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Chaotic Sound Synthesis

Solutions of chaotic and fractal equations have provided artistically new and interesting classes of images and sounds. Musical scores have been produced from Mandelbrot and Julia set images, and chaotic and fractal techniques have been used to directly synthesize a variety of sound waveforms (see, for example, Monro 1991). One popular approach to sound synthesis is based on Chua's circuit (Mayer-Kress et al. 1993; Mayer-Kress, Choi, and Bargar 1993; Hunt and Johnson 1993). Another method, described here, uses cross-coupled frequency-modulated oscillators (chaotic FM synthesis). Sounds produced by the chaotic FM technique vary from pure sine waves to a variety of complex signals and colored noises. The chaotic FM algorithm includes conventional FM synthesis as a subset. This chaotic FM method can be combined with other chaotic synthesis methods, such as the Ueda attractor, providing an even wider range of sounds. Chaotic synthesis and filtering can be used in both the analog and digital domains, providing similar results. Several examples of chaotic synthesis methods in both domains are provided in this article. The emphasis, however, is on the analog methods, with examples using the Moog modular synthesizer, Buchla electronic musical instruments, and electronic analog computers.

Background

Chaotic systems are deterministic dynamic systems that have a high sensitivity to initial conditions. Only dynamic systems that include a nonlinear feedback path are capable of chaotic behavior. Common examples of chaotic systems include coupled pendulums, pseudorandom number generators, and the earth's weather system. Musical examples include sounds produced by brass and reed instruments when played in a certain manner, and the sounds produced by an electric guitar in heavy feedback. With a small change in an initial value, the system output rapidly diverges in an unpredictable manner. A measure of this chaotic sensitivity to initial conditions is the Lyapunov exponent, one of which has a positive value in a chaotic system (Moon 1992; Peitgen, Jurgens, and Saupet 1992). The maximal Lyapunov exponent is a time-average logarithmic measure of the rate of divergence of nearby trajectories, and it takes on a positive value if the system is chaotic.

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Electronic Analog Computers

Electronic analog computers provide a powerful tool for chaotic signal generation and processing. As most musicians are not familiar with analog computers, they are briefly described here. Electronic analog computers were originally developed for a variety of aerospace applications, including real-time aircraft and missile flight-dynamics simulation (Korn and Korn 1972). Analog computers provided a powerful method for solving nonlinear differential equations, and were instrumental in the development of chaotic systems theory (Gleick 1987). The electronic analog computer is in many ways the aerospace equivalent of the analog modular music synthesizer. This type of computer consists of a modular patchable arrangement of operational amplifiers, integrators, summers, multipliers, diode function generators, and other elements (Korn and Korn1972). Electronic analog computers were used extensively from 1950–1980, similar to the time in which analog electronic musical instruments were popular. The analog computer has been replaced in virtually all applications by the digital computer and, like the analog modular synthesizer, is now considered obsolete. Several representative electronic analog computer models are listed in Table 1.

Table 1. Several small analog computers suitable for electronic music applications

Model	*Operational Amplifiers*	*Multipliers*	*Function Generators*	*Notes*
Comdyna GP-6	8	2	Optional	Desktop
Comdyna 808	8	4	Optional	Rack-mount
EAI TR-20	20	3	2	Table-top
EAI TR-48	48	5	8	Table-top

With the recent renewed interest in analog modular musical instruments, the analog computer may again become a useful tool. Many analog computers use a ±10-V signal voltage range, and are electrically compatible with Moog, Buchla, and other analog musical instruments. Many types of musically useful processing that can be done by an analog computer have no equivalent in an analog modular synthesizer. The electronic analog computer is capable of many operations, ranging from simple control-voltage scaling and summing to complex fractal/chaotic sound synthesis and processing.

When programmed with chaotic patches, the electronic analog computer can be interconnected with electronic musical instruments (see Figure 1), providing amplitude-, waveform-, and frequency-dependent nonlinear filters. Unlike conventional filters, these chaotic filters are sensitive to the input signal's amplitude and waveform, and can produce frequencies not present at the filter input. The sounds produced by these types of filters are unique and musically interesting. The analog computer can be replaced by simple analog operational amplifier circuits, providing similar results at the loss of some versatility. These analog circuits produce the classic chaotic results, i.e., high sensitivity to initial conditions, period doubling, broad spectral outputs, transient chaotic operation, etc. External signals can be injected into the circuits, allowing an unusual type of nonlinear amplitude/frequency filtering. Analog chaotic circuits are useful for sound generation, sound filtering, control-voltage generation, and control-voltage processing in analog modular synthesizers.

Ueda Attractor

Chaotic signals can be produced as the solution of a set of nonlinear differential equations—for example, the Lorenz or Duffing equations (Moon 1992). The Ueda attractor, a simplified form of the Duffing equation, is a second-order nonlinear differential equation that can be readily solved in a small analog computer (see Figure 2):

$$\ddot{x} + k\dot{x} + x^3 = B \cos(\omega t) \quad (1)$$

The Logistic Equation

Similar chaotic results can be obtained in digital computers. One of the simplest and most-studied chaotic systems is a first-order nonlinear difference equation called the Logistic equation:

$$x_{n+1} = \lambda (x_n - x_n^2). \quad (2)$$

This equation states that the next x value will be a coefficient λ times the difference between the current x value and current x^2 value ($0 < x < 1$). This calculation can be easily carried out on a pocket calculator. The solution to this equation is chaotic when the value of λ is between 3.57 and 4. A common application of chaotic algorithms in computer science is the maximal-length-sequence pseudorandom noise generator, a binary version of a chaotic system.

Although the Logistic equation is normally programmed in a digital computer, it is a discrete-time equation that can also be solved in analog

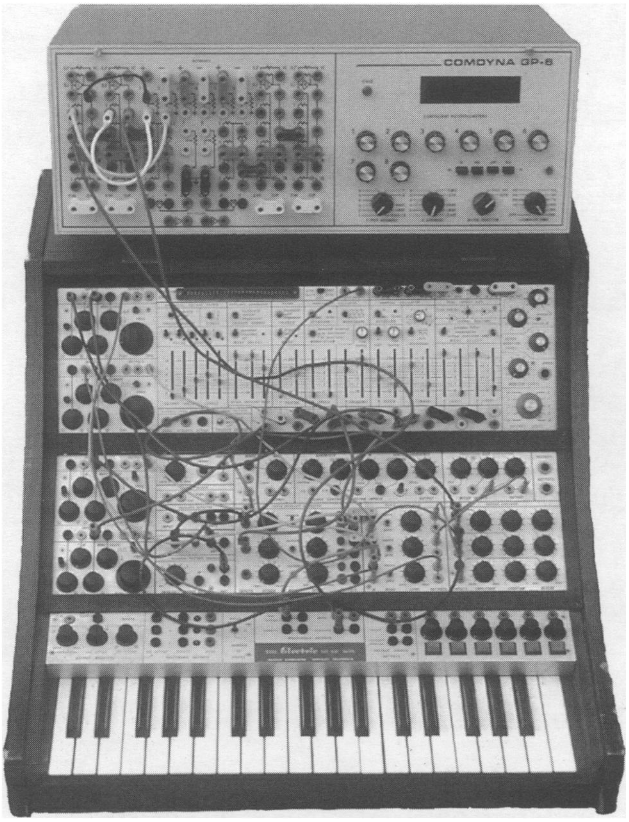


Figure 1. The Comdyna GP-6 analog computer on top of a small Buchla 200 series Electric Music Box. The GP-6 analog computer is programmed with patch cords connected to a patch-board area. Knobs to the right of the patch board are used to set program coefficients, select signal-monitoring points, and select different operating modes. Front-panel displays are used to show potentiometer coefficients, problem results, and fault conditions caused by operational-amplifier overload.

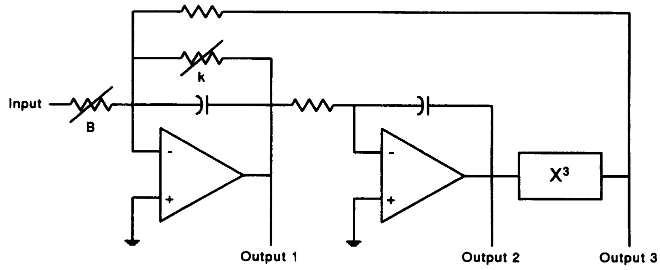


Figure 2. The Ueda attractor is similar to a state variable filter, except that the inverting stage has been replaced with an x^3 circuit and the signal input is connected to the first integrator stage. The Ueda attractor is based on a ring-circuit topology that includes a pair of integrators and an x^3 function. A chaotic system requires both feedback and nonlinearity. The feedback is provided by the ring-circuit topology, and the nonlinearity is provided by the x^3 circuit. The x^3 circuit can be constructed from a pair of analog multipliers.

form. As an example, the Logistic equation was programmed into an analog Moog modular music synthesizer. Three 902 VCA modules and a 928 sample-and-hold module were interconnected as shown in Figure 3. A bifurcation plot of the Logistic equation is shown in Figure 4.

Hyperchaotic Systems

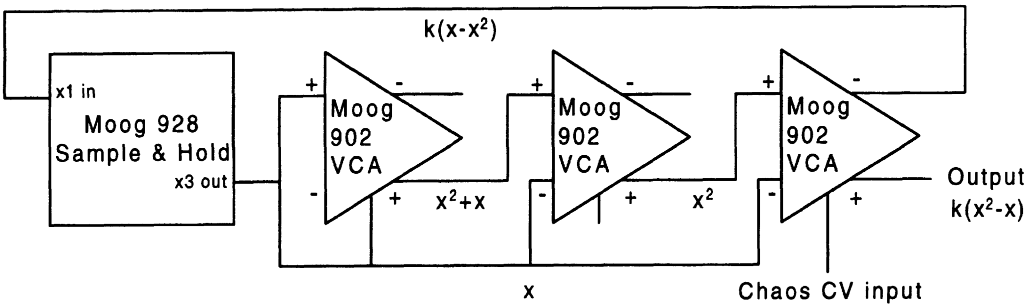
When multiple chaotic interactions in a multidimensional phase space occur, the system can become hyperchaotic (Moon 1992). Many real-world systems are likely capable of this behavior, although relatively few have been studied. Hyperchaos requires, at a minimum, a four-dimensional system with two or more positive Lyapunov exponents (Rossler 1979). In hyperchaotic systems, the phase space can be stretched in a multiplicity of directions. One hyperchaotic system that has been studied numerically is a pair of cross-coupled Van der Pol oscillators (Kapitaniak and Steeb 1991). A variation of this technique is chaotic FM synthesis.

Chaotic FM Synthesis

Around 1970, I composed a 20-min piece of experimental music on a small Buchla Music System 3 (MS3) instrument. This piece of music was a complex and evolving sound sculpture. The small Buchla electronic musical instrument included two voltage-controlled oscillators (VCOs), a programmable pulse generator, three envelope generators, and an envelope follower.

After storing the tape for many years, I listened to it again. Even though the Buchla MS3 had only two audio oscillators, the piece of music had an unexpectedly wide range of complex sound textures. The sonic timbres produced by the two oscillators varied continuously over a wide range from pure sine waves to white noise, and often had interesting stereophonic properties. Furthermore, it did not seem possible to approximate this piece of electronic music either by synthesis or by

Figure 3. An analog implementation of the Logistic equation on a Moog modular music synthesizer. The 902 VCA module is a voltage-controlled, variable-gain amplifier with both differential inputs and outputs. The – and + signs indicate inverting and noninverting inputs (or outputs). Two 902 VCA modules are used to form the x^2 term. A third 902 VCA module is used to form the $k(x - x^2)$ term. The control-voltage input into the third 902 VCA is the chaos level (k). All 902 modules were set to the linear voltage-control mode. The 928 module provides a one-sample time delay. An internal oscillator in the 928 module sets the sample interval. Adjusting the 928 slew rate limiter distorts the discrete-time nature of the system, providing interesting variations to the chaotic output.



sampling on a state-of-the-art Kurzweil K2000 music synthesizer. It rapidly became apparent that chaotic dynamics were at the core of this musical piece.

The foundation of the musical piece was a set of cross-coupled FM-modulated-oscillators: VCO 1 was frequency-modulating VCO 2; and likewise, VCO 2 was frequency-modulating VCO 1 (see Figure 5). As the FM gains were increased beyond a certain point, the sounds became clearly chaotic as each oscillator was frequency modulating the other. With increasing FM gains, the sound assumed a wide variety of timbres, finally becoming chaotic and eventually turning into white noise. Additionally, the Buchla oscillator frequencies and waveforms could be varied, providing an even larger sonic palette. The patch was further musically enhanced by a feedback loop through the envelope follower, programmable pulse generator, and envelope generator.

As an experiment, the two-oscillator chaotic FM synthesis portion of the patch was programmed in an IBM PC computer. The sounds produced in this experiment were quite similar to those produced by the Buchla 258-oscillator module. The following pair of difference equations were used to implement a set of chaotically coupled linear-frequency-modulated oscillators operating at a 44.1-KHz sample rate.

$$L_{n+1} = L_n e^{-i(k_1 \mathrm{Re}(R_n)+2\pi(f_1/s))} \quad (3a)$$

$$R_{n+1} = R_n e^{-i(k_2 \mathrm{Re}(L_n)+2\pi(f_2/s))} \quad (3b)$$

where:

$L_0 = R_0 = 1$ = initial oscillator value (complex signal vector),

$f_1 = -4{,}050T$ = frequency of oscillator 1 (Hz),

$f_2 = 800T + 200$ = frequency of oscillator 2 (Hz),

$k_1 = 10{,}000T$ = oscillator 2 FM coupling to oscillator 1,

$k_2 = 20{,}000T$ = oscillator 1 FM coupling t oscillator 2,

$s = 44{,}100$ = sample rate (Hz),

T = independent control variable (dimensionless), and

$i = \sqrt{-1}$.

The stereo audio output was produced from the real component of the L (left-channel) and R (right-channel) signals.

Nonlinearity and feedback are necessary conditions for the existence of chaotic processes. In this pair of equations, feedback is produced by cross-coupling the difference equations (Equations 3a and 3b) through the k_1 and k_2 terms. The required nonlinearity is developed through the real-part (Re) functions of the oscillator output. For a unit-length signal vector, the k_1 and k_2 terms have nonlinearity gain function of radians/[cos (radians)]. If either k_1 or k_2 has a value of zero, the feedback path is removed, and the equations form a single nonchaotic two-oscillator FM-synthesis operator. If both k_1 and k_2 are set to zero, both oscillators will produce unrelated sine waves.

Figure 4. This bifurcation plot (Gleick 1987) of the Logistic equation was produced directly by a Moog modular music synthesizer coupled to a Tektronix 466 oscilloscope. The vertical plot axis is the output of the Logistic equation patch, shown in Figure 3. A Moog 921 oscillator with a sawtooth output was used to sweep the value of λ, providing the horizontal plot axis. The bifurcation plot provides an insight into the waveforms produced by the Logistic equation. When λ is low in value, the circuit output is a square wave at ½ of the sampling frequency. As λ is increased in value, the control-voltage output slowly bifurcates into a four-level waveform, then into eight levels, and finally into random voltage levels. Similar techniques can be used to produce Poincaré maps and other displays of chaos (Gleick 1987). These techniques can be used to produce voltage-controlled bifurcation and Poincaré outputs suitable for controlling musical parameters.

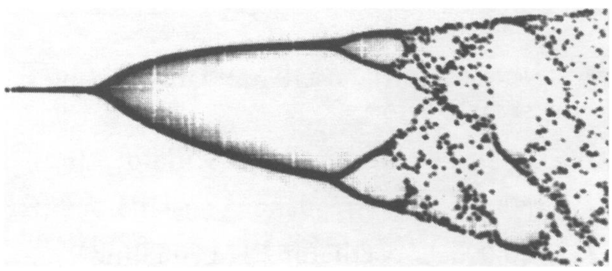
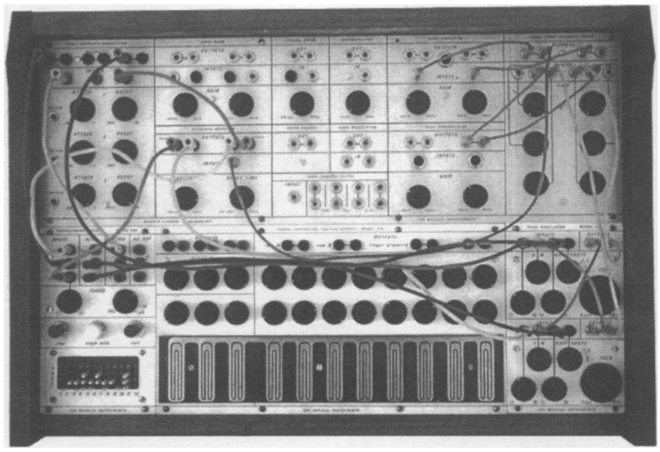


Figure 5. A Buchla Music System 3 with a chaotic patch. The dual oscillator at the lower right has the two outputs cross-coupled to the two FM inputs.



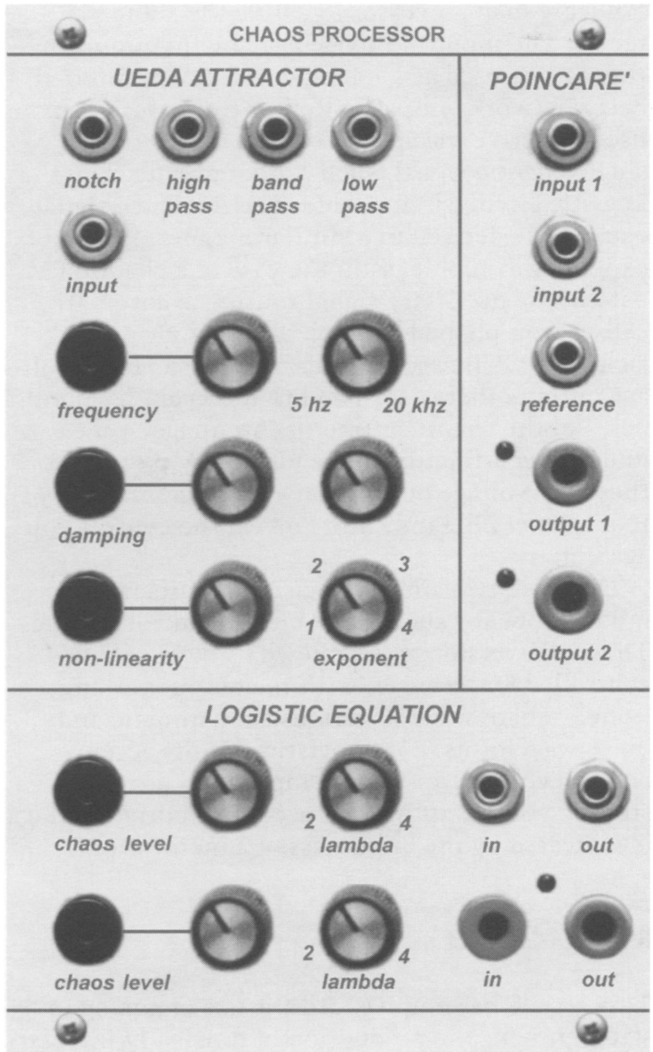
The chaotic signal produced by Equation 3a is shown as a spectrogram on the front cover of this issue. The y-axis has a frequency span from DC to 22.05 kHz. The x-axis corresponds to T, which is linearly swept from 0.468–0.625. This spectrogram shows both regions of wideband noise and regions of complex oscillations with structures somewhat analogous to classical period-doubling areas. Like a maximal length pseudorandom number generator, this time series is completely repeatable if the same kernel, coefficients, and floating-point word format are used.

Probably all modular analog music synthesizers are capable of chaotic FM synthesis. One of the most popular applications of the small EMS AKS briefcase synthesizer is the creation of unusual chirps and noises. These sounds are often based on cross-coupled oscillators that produce chaotic oscillations. Even a pair of cross-coupled frequency-modulated laboratory function generators can produce a wide variety of interesting chaotic FM sounds. The cross-coupling method for producing chaotic signals can be used with modules other than conventional oscillators. For example, when using the Buchla 265 "source-of-uncertainty" module, the random voltage outputs can be cross-coupled to the probable-rate-of-change control inputs. It is hard to describe the result, but it is musically interesting.

In chaotic FM synthesis, as is typical of all chaotic processes, it is desirable to have high-resolution (0.05 percent or better) control of the oscillator frequencies and FM cross-modulation gains. In analog systems, a ten-turn helipot can be used to provide this type of controllability. Even then, significantly different sounds may occur as the pot wiper jumps a single wire turn. Patching tricks can be used in the Buchla 200 to obtain a similar fine chaos control. Analog computers have an excellent ability to provide high-magnification exploration of chaotic systems. Chaotic FM methods, like other chaotic systems, are closely related to fractal structures, which can have exceedingly fine detail. This is rather like looking through a microscope at a large object with fine detail. There is quite a bit of acoustic detail in the chaotic FM algorithm, and one must be careful not to miss the subtleties as the coefficients are varied.

As another experiment, the previously described Ueda attractor was programmed into a Comdyna GP-6 analog computer. The analog computer was used to chaotically modify one feedback path of a

Figure 6. An "artist's conception" of a chaos module for a Buchla 200 series Electric Music Box.



chaotic FM synthesis patch on a Moog analog modular synthesizer. Four audio outputs, suitable for a quadraphonic sound space, were obtained: two sine-wave outputs from the two oscillators and two outputs from the two integrator stages in the Ueda attractor. The sounds produced were clearly chaotic, and formed a superset of sounds relative to those produced separately by either the standard chaotic FM patch or the Ueda attractor. A Poincaré map (Gleick 1987) could be readily formed on a storage oscilloscope, showing the unusual dynamics of the underlying strange attractor. When the analog computer was put in the repetitive-operation mode, interesting rhythms were formed.

A Chaos Module for an Analog Sound Synthesizer

The theory of chaotic dynamics had not been developed during the time that most analog modular synthesizer development occurred (1965–1980). What synthesizer modules would have resulted from a formal knowledge of chaotic dynamics theory? Chaotic sounds are produced by many nonelectronic musical instruments, and the desire to emulate these would likely have driven a desire for a chaos module. The following is a design concept for a chaos module that is compatible with a Buchla series 200 Electric Music Box. The chaos module includes three subsections: an Ueda attractor for audio processing, a Poincaré control-voltage processor, and a pair of Logistic circuits, one for audio signals and one for control-voltage processing. An artist's-concept drawing of this module is shown in Figure 6. The concepts presented here could also be used in the design of an equivalent software module for a digital synthesizer or computer.

The similarities in circuit topology between the Ueda attractor, state-variable filter, and quadrature oscillator suggest a module with multiple capabilities. By using an x^k function, the circuit can be dynamically varied between a Ueda attractor ($k = 3$) and a state-variable filter or quadrature oscillator ($k = 1$). The quadrature-oscillator capability can be obtained by supporting an infinite Q and a soft limiter. Three terms will be voltage-controlled: k, Q, and frequency. There is one audio input and four audio outputs (high pass, low pass, band pass, and band reject). The band-pass and low-pass outputs are in phase quadrature.

The Poincaré map processor consists of a circuit that produces a short pulse at each positive-going zero-crossing of the reference audio input that drives a pair of sample-and-hold circuits. Normally, two of the outputs of the Ueda attractor in phase quadrature would be connected to the

Poincaré map processor to derive the x and y terms. The input to the Ueda attractor circuit in this case is used as a reference input. An internal delay network, all-pass filter, or Dome filter can be used to derive the second channel when only a single chaotic signal, such as that produced by the Logistic circuit, is available. The reference signal can also be derived in a similar manner if it is not explicitly available, as in the case of a chaotic FM patch. The necessary signal routing is automatically accomplished by "normaled" front-panel jacks. The Poincaré function provides control-voltage outputs that correspond to the x- and y-coordinates of the chaotic attractor. An image of the underlying attractor can be formed by plotting these x-y voltage outputs on a storage oscilloscope. Front-panel LED indicators reveal the control-voltage outputs.

There are two similar Logistic circuits in this module: one for audio, and one for control voltages. The audio version acts somewhat like a voltage-controlled waveshaper, with the output varying from a square wave to various subharmonic and noise waveforms. Both Logistic circuits accept chaos-level control-voltage inputs and provide stepped voltage outputs. The control-output voltage is indicated by the brightness of a monitor LED.

Analog Methods

This article has emphasized the use of analog instruments for the production of musically interesting chaotic systems. While the current-generation digital synthesizers are very capable of emulating conventional musical instruments, they are more limited in the exploration of abstract musical ideas. Here, the older analog modular instruments provide two significant advantages: (1) the modular analog systems can be patched into a wide range of nonstandard topologies, including various classes of chaotic systems; and (2) modular analog systems provide multidimensional wide-range, wide-bandwidth, high-resolution (continuous) parameter control.

Chaotic signals—such as those produced by chaotic FM synthesis methods, the discrete-time Logistic equation, and related bifurcation displays—can be directly produced by a conventional, unmodified, analog modular music synthesizer. Almost all currently existing digital musical instruments allow only a narrowly defined set of preconceived topologies (patches). As an example, virtually all current digital synthesizers exclude nonlinear feedback paths, a mandatory requirement for the existence of a chaotic system. One significant exception is Kyma, which is capable of a wide variety of user-defined patches, including those with nonlinear feedback.

Wide-band, wide-range, high-resolution, continuous parameter control is desirable because of the fractal nature of the chaotic systems. These types of control signals are produced naturally by analog modular synthesizers and electronic analog computers. Most currently available digital systems are quite limited in producing these types of control signals. This is not to say that digital technology is incapable of solving these limitations. As was shown earlier in this article, chaotic algorithms are easy to implement in digital form.

Conclusions

Chaotic systems are useful for sound generation, sound filtering, control generation, and control processing in electronic musical instruments. A chaotic system fundamentally requires that a nonlinear feedback loop be present. Chaotic processes can be implemented equivalently in analog and digital systems.

Chaotic FM synthesis produced by cross-coupled frequency-modulated oscillators provides a particularly simple and powerful tool for the production of a wide variety of dynamically variable, musically interesting timbres. This single patch is capable of producing sounds that are dynamically variable, from pure sine waves to various forms of complex colored noise. Chaotic FM synthesis includes conventional FM synthesis methods as a subset. Probably all modular analog synthesizers can use chaotic FM synthesis techniques without significant modifications.

Tools of chaotic dynamics research can also be used in a musical context. Voltage-controlled bifurcation and Poincaré maps can be used to control musical parameters in analog modular synthesizers. Equivalent software programs can be implemented in digitally based music synthesizers (or on general-purpose computers).

Chaotic processes can be combined, providing a larger subset of available sounds. One example is the insertion of the Ueda attractor into the chaos, producing feedback paths of a chaotic FM synthesis patch. The sounds produced are chaotic but have a wider range of sonic characteristics than those produced individually by either chaotic FM synthesis or the Ueda attractor. Hyperchaotic systems have been studied by only a small percentage of the chaotic dynamics researchers. This area is worthy of much more attention. For example, what happens if more than two oscillators are FM-coupled in a ring or star configuration? What happens if other cross-modulation techniques, such as amplitude modulation (AM), double sideband (DSB), and single sideband (SSB) are used? What happens if a pair of Ueda attractors are cross-coupled, or if a reverberator is inserted into a chaotic feedback loop?

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